25047275

QP CODE: 25047275

Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2025

Third Semester

M Sc MATHEMATICS

Core Course - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

2019 ADMISSION ONWARDS

321E6FDE

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 9x, 0 < x < 2\pi$.
2. Define the inetgral transform of f. Also define the Exponential Fourier transform, Laplace transform and Mellin transform.
3. Define a linear function. If $\mathbf{f} : \mathbf{R}^n \to \mathbf{R}^m$ is linear then show that $\mathbf{f}'(\mathbf{c}, \mathbf{u}) = \mathbf{f}(\mathbf{u})$ for every $\mathbf{c}$ and every $\mathbf{u}$.
4. Define directional derivative and total derivative. How is the total derivative expressed as sum of partial derivatives?
5. State a necessary but not sufficient condition for differentiability of a complex-valued function of a complex variable $f = u + iv$.
6. Define Jacobian determinant and find the Jacobian determinant for the function $f(z) = 9z + 2z^2$
7. Define open mapping. State and prove a sufficient condition for a mapping to carry open sets onto open sets.
8. Define a Stationary point and a Saddle point.
9. *Find the matrix $[\alpha_{ij}]$ of $G'(a)$ where G is a primitive mapping.*
10. Write two elementary properties of k- forms.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let f be a real valued and continuous function on $[a, b]$. Then for every $\epsilon > 0$, there is a polynomial p, such that $|f(x) - p(x)| < \epsilon$ for every x in $[a, b]$.
12. Prove that the convolution integral $h(x) = \int_{-\infty}^{\infty} f(t)g(x - t)dt$ exists for every x in R, If $f \in L(R), g \in L(R)$ and that either f or g is bounded on R. Moreover if the bounded function f or g is continuous on R. Then h is also continuous on R and $h \in L(R)$.

13. a. Define Jacobian matrix.
b. Find the Jacobian matrix of the transformation $(\rho, \theta, \phi) \leftarrow (x, y, z)$ given by $x = \rho cos\theta sin\phi, y = \rho sin\theta sin\phi, z = \rho cos\phi.$

14. a. State and prove mean value theorem.
b. If f is a real valued function on an open convex subset S of $\mathbf{R}^n$ prove that $f(\mathbf{y}) - f(\mathbf{x}) = \nabla f(\mathbf{z}).(\mathbf{y} - \mathbf{x})$ for $\mathbf{z}$ in the line segment joining $\mathbf{x}$ and $\mathbf{y} \in \mathbf{S}$.

15. For some integer $n \geq 1$, let f have a continuous n^{th} derivative in the open interval (a, b) . Also for some interior point c in (a, b) , $f'(c) = f''(c) = \ldots = f^{n-1}(c) = 0$ but $f^n(c) \neq 0$. Prove that f has a local maximum at c if n is even and $f^n(c) < 0$ and a local minimum at c if n is even and $f^n(c) > 0$ and if n is odd f cannot have a local maximum or local minimum at c.

16. (a) State second derivative test for extrema.
(b) Find the saddle point of the function $f(x, y) = x^2 - 4xy + y^2 + 6y + 2$

17. *Define support of a function f on R^k. Also show that for every $f \in C(I^k), L(f) = L'(f)$.*

18. *Let γ be a $1-$surface in R^3 with parameter domain $[0, 1]$ and $\omega = xdy + ydx$. Then prove that $\int_\gamma \omega$ depends only on the endpoint of the curve γ*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. If $\mathcal{F}(f)$ denotes the Fourier Transform of f, then prove that $\mathcal{F}(f * g) = \mathcal{F}(f).\mathcal{F}(g)$.

20. Show that the composition of two differentiable functions is again differentiable. Also express total derivative of the composite function in terms of total derivatives of individual functions.

21. If both partial derivatives $D_r f$ and $D_k f$ exist in an n- ball $B(c, \delta)$ and if both are differentiable at c then Prove that $D_{r,k} f(c) = D_{k,r} f(c)$

22. *Suppose F is a C' mapping of an open set $E \subset R^n$ into R^n, $0 \in E$, $F(0) = 0$ and $F'(0)$ is invertible. Then there is a nbd of 0 in R^n in which $F(x) = B_1 B_2 \ldots B_{n-1} G_n \circ \ldots \circ_1 G(x)$.*

(2×5=10 weightage)